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AUTHOR	Dias, Ana Lucia Braz
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ABSTRACT

This paper describes student difficulties in first-year calculus classes and suggests some instructional strategies to address these difficulties. Participants in this study were 200 low socio-economic background freshmen majoring in business in Brazil. Results show that the main difficulties are related to lack of understanding of algebraic notation and graphs, lack of understanding of continuum, inability to interpret word problems, and confusion caused by the usual meaning of a word when its mathematical meaning is different. To overcome these difficulties, suggested initial strategies include explaining the idea of the coordinate plane and ordered pairs, helping students understand variables and graphing, teaching about mathematical modeling, and using appropriate textbooks in teaching the graphic aspect of concepts and algebraic notation. The paper includes several illustrated examples of problems together with teaching notes. (KHR)

Overcoming Algebraic and Graphic Difficulties

Ana Lúcia Braz Dias
Universidade Católica de Brasília, Brazil

How does one manage to teach 200 freshmen first-year calculus if they have difficulties with basic algebra and graphs? Teaching Business Majors at Universidade Católica de Brasília, a school that gets its students from the low socio-economic portion of the population of Brazil's Federal District, I have been trying to find this out, and will share with readers some of the difficulties I have found and some of my thoughts on the solutions.

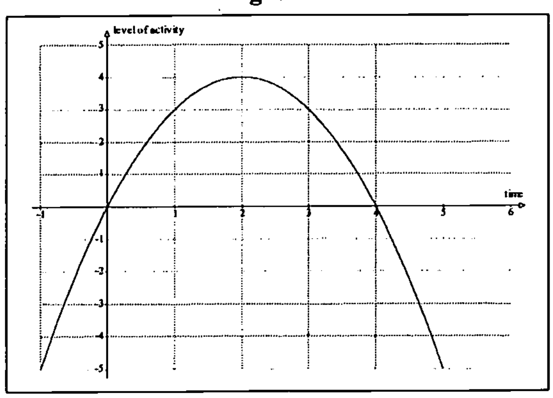
Difficulties with Algebra

Let F: $A \rightarrow B$, $x \in A$, $y \in B$. This is the kind of language usually found in the books. Students are expected to read or understand "let F be a function going from set A to set B, x a variable whose domain is set A and y a variable whose range is set B." Students, on the other hand, ask me, when faced with "f(4)" on a text: "What is this f in front of the 4? Can I just disregard it?"

Difficulties with Graphs

Graphically, given a graph of a function F like that in Figure 1, students cannot tell what the image of three by the function F is.

Figure 1



Students also usually do not have an understanding of the continuum. For example, even after being instructed about the difference between having graphs composed of discrete points and graphs which are lines, very few students could answer the following question in terms of intervals:

What values of the variable *time* have positive images? (See Figure 1.) Usual answers were "1, 2, and 3" or "0, 1, 2, 3, 4." When prompted about there existing numbers between 1 and 2 with images on the graph, a student answered: "Oh, yes, 1/2." I insisted "Only half? No other numbers?" She timidly jotted a 1/4 on the paper.

Lack of Meaning

The most popular textbooks used for this course bring problems that bring up situations such as: "The number of members in an association is given by $f(x) = 100(2x^3 - 45x^2 + 264x)$." The students question how the number of members in an association can be given by such a function. When a problem says "The daily production of a certain factory is $Q(L) = 20{,}000\ L^{1/2}$," the students ask whether they are going to be given such formulas when they start working. The reason for this lack of meaning on the majority of textbook word problems is that there is no effort to acquaint students with the modeling process. Learning how mathematics is really used through modeling is the only way students will be able to use it in their careers later on.

Problems with the natural language also exist, and they are of two kinds: inability to interpret word problems and confusions caused by the usual meaning of a word when its mathematical meaning is different. For example, the word used for slope in mathematics in Portuguese is *declive*. However, in everyday language this word means what in mathematics we would call a negative slope. So every time I said "slope" and pointed to a positive slope on a graph I heard whispers and confusion among the students. One of them finally said she was seeing no slope. She was seeing an *aclive* ("positive slope," in everyday language).

Approaches to Improve the Situation

Syncopated Algebra
To overcome the difficulties with algebraic notation, groundwork had to be done to let students understand what a variable is. They all had used variables procedurally since elementary school and basically think that "variables are letters." If the students do not know that variables may be other symbols and that not all letters are variables, and, more importantly, if they do not know when to create and use a variable, they will not do mathematics.

At our course at UCB's business school I have been emphasizing that variables are symbols chosen to represent any one generic element in a set, the domain of the variable. I have worked with the students using real data from surveys or research found on the Intemet. I try to bring from real research variables that are not "a letter" but words or expressions, to avoid that habit of specially using x's and y's. So they work with variables such as SEX, DEGREE, WRKSTATUS, and can have ordered pairs such as (SEX, DEGREE) instead of the usual (x, y). I also have them collect their own data and create variables, specifying their "names"and their domains, and to form general sentences about the elements of their sets of data.

Using syncopated algebra is also helpful. As much as possible, I write, for example, "the image of 4 by the function G" instead of g(4) for a while until they are tired of it and accept and understand the g(4). Or

$$\frac{\text{resultant variation on the dependent variable}}{\text{variation occurred on the independent variable}} \text{ instead of } \frac{\Delta y}{\Delta x}$$

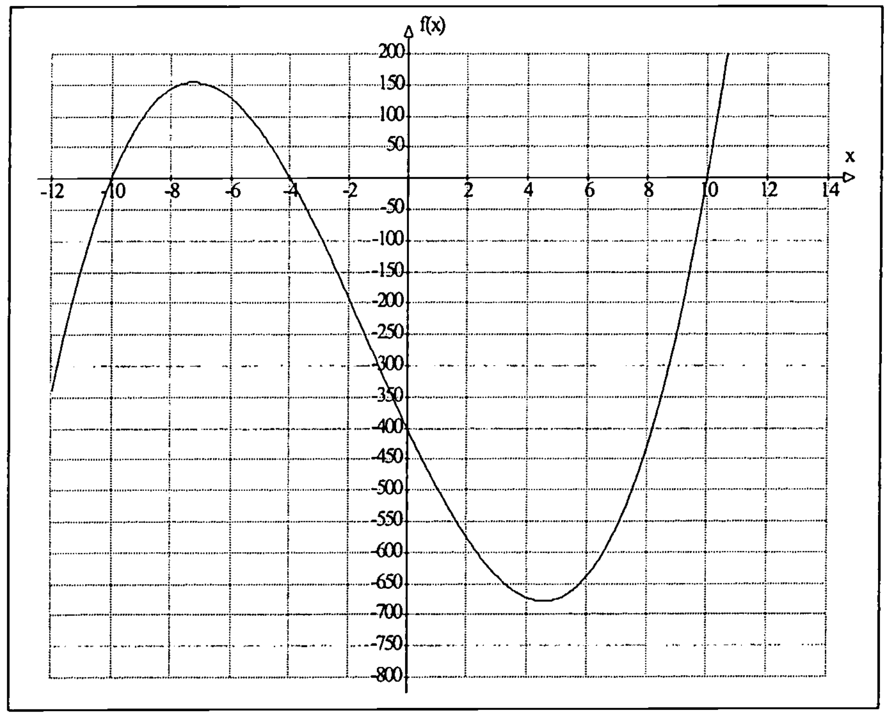
Lots of Work with Graphs
Facing the students' difficulties, initial work had to be done with graphs, starting by explaining the whole idea of algebrizing the plane and how any point on the plane can be determined by an ordered pair. This of course has to be preceded by an understanding of the "Real line." Students bring these ideas vaguely from high school, but, as said before, they lack conceptual understanding. They all think they can plot a point given the coordinates, and they do when the coordinates are (1,2), but give them the graph of a function F and ask them to point to (2, f(2)) and the difficulties start showing. As for the real numbers, as mentioned before they carry ideas such as that there is a finite number of numbers between two integers or that there is "a next real number after 1.1," for example.

Working on simple exercises such as the one in Figure 2 is a situation for a lot of learning.

Figure 2

Plot on the Cartesian plane below the following points:

(2, 50)
(4, -200)
(4, f(4))
(-8, f(-8))
(2, f(-8))
(6, - f(-8))



Students had a lot of difficulty in seeing that the distance from a point on the horizontal axis and its image on a graph can be the ordinate of another point, in another location of the horizontal axis. Transferring lengths and using them as coordinates of different points was a good exercise.

The students have to start viewing a coordinate as the measure of a segment. Understanding how the real line was constructed, based on a chosen unit, and that for each length of line segment a real number is associated (that is, for each point a real number is associated) is essential. Work like this may be too rigorous and formal, but how to even have the procedural knowledge the textbooks emphasize without conceptual knowledge?

This exercise showed students that their previous knowledge on points and coordinates was not as well established as they thought. Apparently they had learned "Go to the horizontal axis to mark the first coordinate and to the vertical axis to mark the second coordinate." However, when they had to mark a point such as (2, f(-8)), they had to:

1. Go to the -8 on the horizontal axis.
2. Measure the length f(-8) above (or below) it until the curve of the graph of F.
3. Transfer that length to the 2 on the horizontal axis.

Many of the students just could not agree with this. They said: "To mark the second coordinate I must go to the vertical axis, and here am I going to the horizontal axis!"

Students finally became accustomed to the idea of coordinates when we started writing them as (side distance, height). For example, a point (4, 5) is 4 units distant from the vertical axis to the right, and five units high from

the horizontal axis. This also helped a lot with the sign of quantities, because sometimes they would not admit that f(-8) or that -f(8) could be positive. When we started talking about "positive height," "negative height," "units to the left" or "to the right of the vertical axis," and other strange things that for us made sense, things started to get clearer.

Modeling

In different semesters I have tried different approaches to deal with students' lack of knowledge of the modeling process. I have tried to *teach about* modeling, what it means and its steps, showing examples, and I have tried to have students model real situations. While the second alternative is more difficult to work with, it is much more engaging for students. While we naturally cannot expect students to create great models at this stage, they can at least experiment with each step of the process, and with the help of a curve fitter, make simplifications and approximations to real situations, such as forecasting of maxima and minima or future values using the functions chosen.

Understanding the difference between the Rationals and the Continuum brings in a good discussion on modeling and the difference between discrete and continuous variables. The students become much more critical of all the theory they are learning when they think of which of the measurable variables in the real world are continuous or when they can be assumed continuous.

Textbooks

This year I was particularly happy to find that one of Deborah Hughes-Hallett's calculus books (Hughes-Hallett et al., 1994) had been translated into Portuguese. This book addresses a lot of these same ideas, such as the intense work with the graphic aspect of every concept and the easiness with the algebraic notation. One can find there, for example, $\frac{\text{vertical increment}}{\text{horizontal increment}}$ (my re-translation into English), instead of $\frac{\Delta y}{\Delta x}$. I promptly adopted it as a textbook (although still doing original side work, especially in modeling and curve fitting with software), but found that it is still too complex for my students. To really study deeply the first chapter of the book I might need a whole semester, so I have to help my students go through the book, leaving a lot of good things behind.

To give the readers an idea, two of the simplest exercises in Hughes-Hallett's book, in Figures 3 and 4 (Hughes-Hallett et al., 1997, pp. 23-24, my re-translation), gave rise to the long, step by step worksheet I had to write for the students (Figure 5). They had trouble seeing the relationship between the curvature of a graph and the sign (positive or negative) of the variation of the dependent variable, especially the case of decreasing functions. Specifically, they would think that greater and greater "falls" on a decreasing graph (downward curvature) meant that the variation on the dependent variable was increasing (suffering a positive variation).

Figure 3

- Each function in table 1.10 is increasing, but each increases in a different way. Which of the graphs of figure1.24 best fits each function?

t	g(t)	h(t)	k(t)
1	23	10	2.2
2	24	20	2.5
3	26	29	2.8
4	29	37	3.1
5	33	44	3.4
6	38	50	3.7

Table 1.10

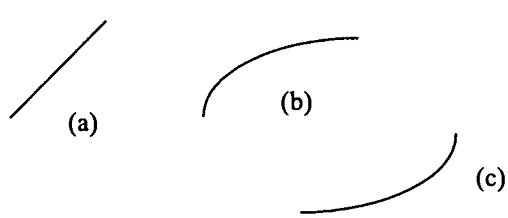


figure 1.24

Figure 4

Each function in table 1.11 is decreasing, but each decreases in a different way. Which of the graphs of the figure1.25 best fits each function?

x	f(x)	g(x)	h(x)
1	100	22.0	9.3
2	90	21.4	9.1
3	81	20.8	8.8
4	73	20.2	8.4
5	66	19.6	7.9
6	60	19.0	7.3

Table 1.11

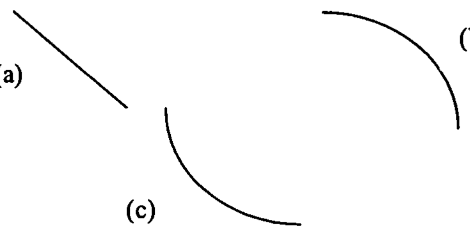
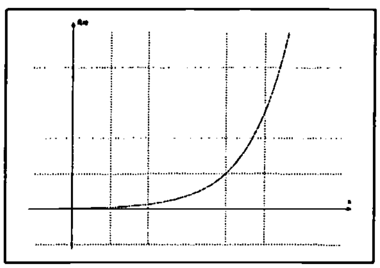


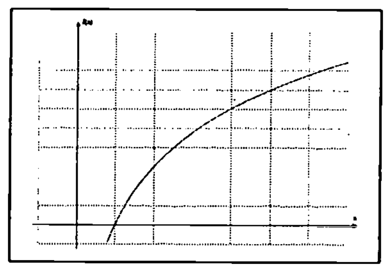
Figure 1.25

Figure 5
Worksheet

First, let us examine the graphs below:



Graph 1



Graph 2

In both graphs, the dependent variable increases as the independent variable grows.

As for the **variation on the dependent variable** for a given **variation on the independent variable** (use the grid to examine intervals of the same length on the horizontal axis), as the independent variable increases:

The variation of y ________________ on graph 1 (increases or decreases?)
The variation of y ________________ on graph 2. (increases or decreases?)

We see then that, when a graph is **concave downward**, as the independent variable increases, the variation on the dependent variable for a given variation on the independent variable, ____________ (increases or decreases)?
And when a graph is **concave upward**, as the independent variable increases, the variation on the dependent variable for a given variation on the independent variable, ____________ (increases or decreases)?

Save the conclusion you got above (let us call it conclusion (*)) because we will compare it with what you conclude later.

Let us examine now the tables in exercise 5:

The table below gives us the values of h(t) for the integer values of t from 1 to 6. In the third column, we started to calculate the variation that h(t) suffered when t varied by 1 unit, from the previous line to the line in question. Continue to fill out that last column:

t	h(t)	variation on h(t)	
1	10		
2	20	(from t=1 to t=2)	10
3	29	(from t=2 to t=3)	
4	37	(from t=3 to t=4)	
5	44	(from t=4 to t=5)	
6	50	(from t=5 to t=6)	

Do the same in the following table, with the values for g(t) (Calculate the variation of g(t) from one line to the next):

t	g(t)	variation on g(t)	
1	23		
2	24	(from t=1 to t=2)	1
3	26	(from t=2 to t=3)	
4	29	(from t=3 to t=4)	
5	33	(from t=4 to t=5)	
6	38	(from t=5 to t=6)	

Finally, do the same in the following table, with the values for k(t) (Calculate the variation of k(t) from one line to the next):

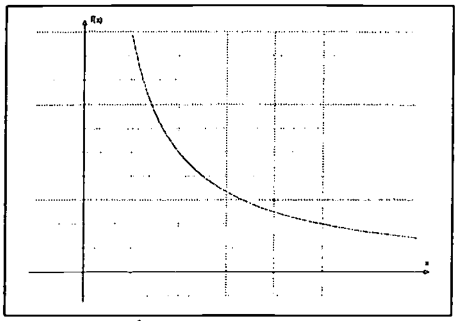
t	k(t)	variation on k(t)	
1	23		
2	24	(from t=1 to t=2)	0.3
3	26	(from t=2 to t=3)	
4	29	(from t=3 to t=4)	
5	33	(from t=4 to t=5)	
6	38	(from t=5 to t=6)	

We see that, as t increases, h(t), g(t), and k(t) increase. However, as t increases, h(t) **increases faster or slower** every time? ________________ So, how would its graph look? Concave upward or concave downward? ________________

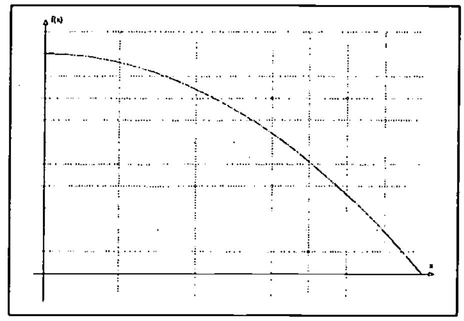
What about g(t), does it increase faster or slower every time? ___________ So, how would its graph look? Concave upward or concave downward? ________________

And what do you say about k(t)? ___________________________

Let us now examine decreasing functions:



Graph 3



Graph 4

In both graphs, as the independent variable increases, the dependent variable decreases. But does it decrease slower or faster every time?

In the graph that is concave upward, the dependent variable decreases _________ (slower or faster)?

In the graph that is concave downward, the dependent variable decreases _________ (slower or faster)?

Now let us examine the tables in exercise 6:

x	f(x)	variation on f(x)
1	100	
2	90	(from t=1 to t=2) **-10**
3	81	(from t=2 to t=3)
4	73	(from t=3 to t=4)
5	66	(from t=4 to t=5)
6	60	(from t=5 to t=6)

We see that, as x increases, f(x) decreases. But does it **decrease faster every time or slower every time?** ________________

So, how would its graph look? Decreasing and "falling" greater and greater distances or falling smaller and smaller distances? That is, would it be **concave downward or upward**? ________________

Now, in terms of the **variation of f(x)**, as x increases, is the variation of f(x) increasing or decreasing? (See the values with which you completed the table and answer. But pay attention! We are dealing with negative numbers. -10 is greater or smaller than -6? So the **variation of f(x)** is increasing or decreasing?) ____________________

Repeating what you have just written: the graph of F is concave _______ (upward or downward?) and, as x increases, the variation in f(x) for a given variation in x _______ (increases or decreases?).

Now compare this conclusion with the conclusion (*) that you reached in the case of increasing functions. They should be the same (that is, (*) should still work).

Continue the exercise for the functions G and H: (...)

Conclusions

The goals in introductory mathematics courses for non-mathematical careers may vary from having students memorize some rules and procedures (such as rules of derivation and rules for the relationship between curvature of the graph of a function and the sign of the second derivative of the function) to helping them gain insight into the mathematical concepts they will be using. If we have the second goal, more time and effort than usual may be necessary if the students did not have adequate preparation in their previous schooling. The activities described in this workshop may seem condescending and too easy for college students. However, my experience has been that after the hard work students have thrilled at being able to understand the exercises and theoretical parts of the textbooks. They have often made positive manifestations about our work in class and the extra worksheets and asked for more of this kind of guidance.

Reference

Hughes-Hallett, D., et al. (1997). *Cálculo, vol. I.* Rio de Janeiro: Livros Técnicos e Científicos Editora.

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